

Current dependence of spin torque switching rate based on Fokker-Planck approach

Tomohiro Taniguchi and Hiroshi Imamura

National Institute of Advanced Industrial Science and Technology (AIST),
Spintronics Research Center, Tsukuba 305-8568, Japan

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The spin torque switching rate of an in-plane magnetized system in the presence of an applied field is derived by solving the Fokker-Planck equation. It is found that three scaling currents are necessary to describe the current dependence of the switching rate in the low-current limit. The dependences of these scaling currents on the applied field strength are also studied.

Spin torque induced magnetization switching of a nanostructured ferromagnet in the thermally activated region is an important phenomenon for spintronics applications because the thermal stability and the spin torque switching current of the magnetic random access memory (MRAM) can be obtained from its switching probability [1–5]. The experimentally observed switching probability has been analyzed by the formula $P = 1 - e^{-\nu t}$ [6–19], where the switching rate $\nu = fe^{-\Delta}$ consists of attempt frequency f and switching barrier Δ . It has been often assumed that the attempt frequency f is constant (typically 1 GHz [4]), and that the switching barrier is proportional to current as $\Delta = \Delta_0(1 - I/I_c^*)$, where the thermal stability $\Delta_0 = MH_K V / (2k_B T)$ consists of magnetization M , uniaxial anisotropy field along the easy axis H_K , volume of the free layer V , and temperature T . The current is denoted as I while I_c^* is the spin torque switching current at zero temperature.

However, our recent works revealed the limitation of the applicability of the previous theories [15, 17, 18, 20]. For example, the value of the attempt frequency depends on the current magnitude. Also, the linear scaling of the switching barrier Δ is valid only for $I < I_c$, while Δ depends on the current nonlinearly for $I_c \leq I < I_c^*$, where $I_c (< I_c^*)$ is a characteristic current of the instability of the equilibrium state. The formula in Refs. [18, 20] will enable us to evaluate the thermal stability and the switching current with high accuracies. However, Refs. [18, 20] consider only the zero applied field case, while in the experiments the applied field has been often used to quickly observe the switching [1, 2, 4, 5].

In this paper, we derive the theoretical formula of the switching rate of an in-plane magnetized system in the presence of the applied field by applying the mean first passage time approach to the Fokker-Planck equation. We find that in the low-current region ($I < I_c$), the current dependence of the switching rate is characterized by three scaling currents, I_c , \tilde{I}_c , and I_c^* . The applied field dependences of these scaling currents are also studied.

The system we consider is schematically shown in Fig. 1, where the unit vectors pointing in the magnetization directions of the free and the pinned layers are denoted as \mathbf{m} and $\mathbf{n}_p = \mathbf{e}_z$, respectively. The z -axis is parallel to the in-plane easy axis of the free layer while the x -axis is normal to the film-plane. The positive current is defined as the electron flow from the free layer to the pinned

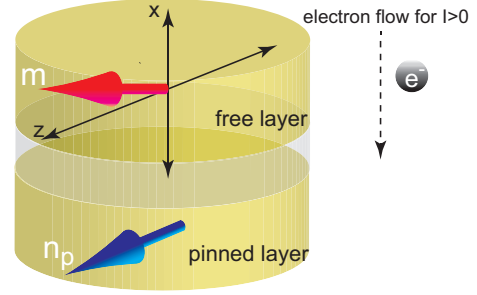


FIG. 1: Schematic view of an in-plane magnetized system.

layer. The energy density of the free layer,

$$E = -MH_{\text{appl}}m_z - \frac{MH_K}{2}m_z^2 + \frac{4\pi M^2}{2}m_x^2, \quad (1)$$

consists of the Zeeman energy, the uniaxial anisotropy energy along the z -axis, and the shape anisotropy along the x -axis, respectively. The minima of the energy density are $\mp MH_{\text{appl}} - (MH_K/2)$, corresponding to $\mathbf{m} = \pm \mathbf{e}_z$, while the energy density at the saddle point, $\mathbf{m} = (0, \pm \sqrt{1 - (H_{\text{appl}}/H_K)^2}, -H_{\text{appl}}/H_K)$, is $E_s = MH_{\text{appl}}^2 / (2H_K)$. Below, the initial state is taken to be $\mathbf{m} = \mathbf{e}_z$. The applied field magnitude $|H_{\text{appl}}|$ should be less than H_K to guarantee two minima of E . The magnetization dynamics is described by the Landau-Lifshitz-Gilbert (LLG) equation,

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H} - \gamma H_s \mathbf{m} \times (\mathbf{n}_p \times \mathbf{m}) + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}, \quad (2)$$

where $\mathbf{H} = -\partial E / \partial (M\mathbf{m})$. The gyromagnetic ratio and the Gilbert damping constant are denoted as γ and α , respectively. The spin torque strength,

$$H_s = \frac{\hbar \eta I}{2eMV}, \quad (3)$$

includes the spin polarization η of the current.

At zero temperature, the initial state becomes unstable when the current magnitude is larger than

$$I_c = \frac{2\alpha eMV}{\hbar \eta} (H_{\text{appl}} + H_K + 2\pi M). \quad (4)$$

The instability of the initial state does not guarantee the switching. The switching at zero temperature occurs when the current magnitude becomes larger than [21]

$$I_c^* = \frac{2\alpha eMV}{\hbar \eta} 4\pi M \frac{\mathcal{N}}{\mathcal{D}}. \quad (5)$$

Here, \mathcal{N} and \mathcal{D} are defined as

$$\mathcal{N} = \sqrt{1+k}[k(1+k)-h^2] \left\{ 2(k^2-h^2)\sqrt{k(1+k)} + h\sqrt{k(k^2-h^2)} \left[\pi + 2\sin^{-1} \left(\frac{h}{\sqrt{k[k(1+k)-h^2]}} \right) \right] \right\}, \quad (6)$$

$$\mathcal{D} = 2h\sqrt{k(1+k)(k^2-h^2)} + k[k(1+k)-h^2]\sqrt{k(1+k)(k^2-h^2)} \times \left[\pi + 2\sin^{-1} \left(\frac{h}{\sqrt{k[k(1+k)-h^2]}} \right) \right], \quad (7)$$

where $h = H_{\text{appl}}/(4\pi M)$ and $k = H_K/(4\pi M)$.

In the thermally activated region $I < I_c^*$, the magnetization dynamics is described by the Fokker-Planck equation, which can be obtained by adding the stochastic torque, $-\gamma \mathbf{m} \times \mathbf{h}$, to the right hand side of Eq. (2) [6], and is given by [9, 22]

$$\frac{\partial \mathcal{P}}{\partial t} + \frac{\partial J}{\partial E} = 0, \quad (8)$$

$$J(E) = \frac{M}{\gamma} \left(\frac{\mathcal{M}_s - \alpha \mathcal{M}_\alpha}{\tau} + \frac{MD\mathcal{M}_\alpha}{\gamma\tau^2} \frac{d\tau}{dE} \right) \mathcal{P} - D \left(\frac{M}{\gamma} \right)^2 \frac{\mathcal{M}_\alpha}{\tau} \frac{\partial \mathcal{P}}{\partial E}, \quad (9)$$

where \mathcal{P} and J are the probability function of the magnetization direction and the probability current, respectively. The diffusion constant $D = \alpha\gamma k_B T/(MV)$ relates to the fluctuation-dissipation theorem as $\langle h_i(t)h_j(t') \rangle = (2D/\gamma^2)\delta_{ij}\delta(t-t')$. The functions $\mathcal{M}_s(E) = \gamma^2 H_s \oint dt [\mathbf{n}_p \cdot \mathbf{H} - (\mathbf{m} \cdot \mathbf{n}_p)(\mathbf{m} \cdot \mathbf{H})]$ and $\mathcal{M}_\alpha(E) = \gamma^2 \oint dt [\mathbf{H}^2 - (\mathbf{m} \cdot \mathbf{H})^2]$ are proportional to the work done by spin torque and the energy dissipation due to the damping on constant energy line, respectively. The precession period on the constant energy line is denoted as τ . Equation (8) describes the Brownian motion of the magnetization in the effective potential \mathcal{E} defined as

$$\mathcal{E} = \int_0^E dE' \left[1 - \frac{\mathcal{M}_s(E')}{\alpha \mathcal{M}_\alpha(E')} \right]. \quad (10)$$

The steady state solution of Eq. (8) is proportional to $e^{-\mathcal{E}V/(k_B T)}$. It should be noted that I_c and I_c^* satisfy $\lim_{E \rightarrow -MH_{\text{appl}} - (MH_K/2)} d\mathcal{E}/dE = 1 - I/I_c$ and $\lim_{E \rightarrow E_s} d\mathcal{E}/dE = 1 - I/I_c^*$, respectively.

The mean first passage time [23], which characterizes how long the magnetization stays in the stable region of the effective potential \mathcal{E} , can be introduced as $\mathcal{T} = \int_0^\infty dt \int_{E^*}^{E_s} dE_1 \mathcal{P}(E_1, t|E^*, 0)$. Here, E^* for $I < I_c$ is the energy density at the initial state, $-MH_{\text{appl}} - (MH_K/2)$,

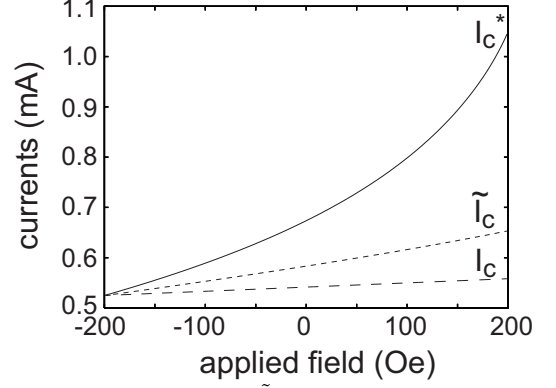


FIG. 2: Dependences of I_c , \tilde{I}_c , and I_c^* on the applied field.

while E^* for $I_c \leq I < I_c^*$ is determined by the condition $\mathcal{M}_s(E^*) = \alpha \mathcal{M}_\alpha(E^*)$. The solution of the mean first passage time is obtained from Eq. (8), and is given by

$$\mathcal{T} = \frac{\gamma V}{\alpha M k_B T} \int_{E^*}^{E_s} dE_1 \int_{E^*}^{E_1} dE_2 \frac{\tau(E_2)}{\mathcal{M}_\alpha(E_1)} \times \exp \left\{ \frac{[\mathcal{E}(E_1) - \mathcal{E}(E_2)]V}{k_B T} \right\}. \quad (11)$$

The switching rate from $\mathbf{m} = \mathbf{e}_z$ to $\mathbf{m} = -\mathbf{e}_z$ is given by $\nu = (1 + I/I_c^*)/(2\mathcal{T})$ [20]. In the low-current region $I < I_c$ and in the high-barrier limit, the switching rate is

$$\nu = \frac{\alpha MV \mathcal{M}_\alpha(E_s)}{2\gamma k_B T \tau(E^*)} \left(1 - \frac{I}{I_c} \right) \left[1 - \left(\frac{I}{I_c^*} \right)^2 \right] \times \exp \left[-\Delta_0 \left(1 + \frac{H_{\text{appl}}}{H_K} \right)^2 \left(1 - \frac{I}{\tilde{I}_c} \right) \right], \quad (12)$$

where $\mathcal{M}_\alpha(E_s) = 8\pi\gamma M \mathcal{N}/[k^2(1+k)^2\sqrt{k^2-h^2}]$ and $\tau(E^*) = 2\pi/[\gamma\sqrt{(H_{\text{appl}} + H_K)(H_{\text{appl}} + H_K + 4\pi M)}]$. The scaling current \tilde{I}_c is defined as

$$\tilde{I}_c = \frac{2\alpha e MV}{\hbar\eta} \frac{4\pi M}{\mathcal{S}}, \quad (13)$$

where the dimensionless quantity \mathcal{S} is defined as

$$\mathcal{S} = \frac{4\pi M \int_{E^*}^{E_s} dE' \mathcal{M}_s(E')/\mathcal{M}_\alpha(E')}{(MH_K/2)(1 + H_{\text{appl}}/H_K)^2 H_s}. \quad (14)$$

As mentioned above, Eq. (12) is valid for $I < I_c$ and $\Delta = \Delta_0(1 + H_{\text{appl}}/H_K)^2(1 - I/\tilde{I}_c) \gg 1$. The attempt frequency is defined as $f = \nu e^\Delta$. On the other hand, in the high-current region $I_c \leq I < I_c^*$, the numerical calculation is necessary to estimate the current dependence of the switching rate ν [20].

Figure 2 shows the dependences of I_c , \tilde{I}_c , and I_c^* on the applied field H_{appl} . The values of the parameters are $M = 1000$ emu/c.c., $H_K = 200$ Oe, $V = \pi \times 80 \times 35 \times 2.5$ nm³, $\alpha = 0.01$, and $\eta = 0.8$, respectively, which are typical values for a magnetic tunnel junction consisting of CoFeB [3, 4, 24, 25]. The scaling current \tilde{I}_c is less than I_c^* , and weakly depends on H_{appl} , compared with I_c^* .

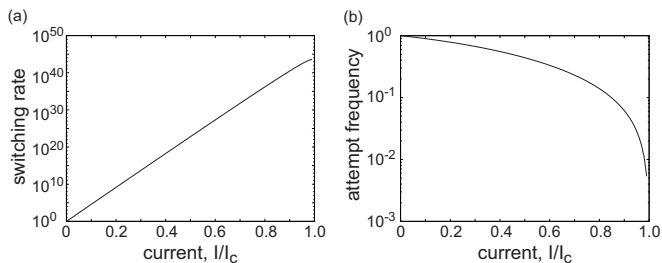


FIG. 3: Dependences of (a) the switching rate ν and (b) the attempt frequency f in the low-current region ($I < I_c$) on the current I for $H_{\text{appl}} = 100$ Oe, where the values of ν and f are normalized by those at $I = 0$ while I is normalized by I_c .

Figures 3 (a) and (b) show the current dependence of the switching rate ν and the attempt frequency f in the low-current region, $I < I_c$. The applied field strength is $H_{\text{appl}} = 100$ Oe. The current dependence of ν in the logarithmic scale is approximately linear due to the linear dependence of the switching barrier Δ , although the attempt frequency f also depends on the current. It should be noted that the scaling current of the switching barrier in the low-current region is \tilde{I}_c , neither I_c nor I_c^* as argued in Refs. [7, 8]. This means that the previous analyses of the experiments [3, 4] underestimate the switching current.

In the above formula, the effect of the field like torque [10] is neglected. On the other hand, recently, the ef-

fect of the field like torque on the relaxation time, $1/\nu$, was experimentally investigated [26], in which the field like torque term is treated as an additional applied field, and is assumed a quadratic function of the bias voltage. From the bias voltage dependence of the relaxation time, the expansion coefficient of the field like torque term was estimated. However, in Ref. [26], the attempt frequency ($1/\tau_0$ in Ref. [26]) is assumed to be independent of the damping constant, temperature, and bias voltage. The combination of our formula developed above with the method in Ref. [26] will help the quantitative estimation of the retention time of MRAM with high accuracy.

In summary, the theoretical formula of the spin torque switching rate of an in-plane magnetized system in the presence of an applied field was derived by solving the Fokker-Planck equation. In the low-current region $I < I_c$, the current dependence of the switching rate is characterized by three scaling currents, I_c , \tilde{I}_c , and I_c^* , where I_c and I_c^* determines the current dependence of the attempt frequency while \tilde{I}_c determines that of the switching barrier. The dependences of these scaling currents on the applied field strength were also studied.

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